JOURNAL OF GEOPHYSICAL RESEARCH VOLUME 65, NO. 5 MAY 1960

The Influence of the Solar Radiation Pressure on the Motion of an Artificial Satellite

PETER MUSEN

Theoretical Division, Goddard Space Flight Center,
National Aeronautics and Space Administration
Washington, D. C.

Abstract. This article reports an investigation of the effect of solar radiation pressure on the motion of an artificial satellite. The theory has been applied to the orbit of the Vanguard I satellite, and is found to produce significant perturbations in the perigee height of that satellite. In the case of a satellite with a large ratio of area to mass the major terms introduced by solar radiation pressure can reduce the perigee distance at the rate of 1 to 2 km per day, so that the lifetime of the satellite will become considerably shorter than it would be without this effect.

Introduction. The perturbing effects of solar radiation pressure on satellite orbits have been generally considered by celestial mechanicians to be negligible. However, the difference between observed and theoretical values of perigee height for the Vanguard I satellite have suggested a re-examination of radiation pressure as a possible source of the discrepancy. We have carried out an investigation of the radiation-pressure perturbation and developed an analytical theory of resonance for the case in which the perigee follows the motion of the sun. It is found that when resonance conditions are nearly satisfied the radiation pressure may produce substantial orbit perturbations over an interval of several months. For a satellite with a large ratio of area to mass the major terms introduced by solar radiation pressure can reduce the perigee distance at the rate of 1 to 2 km per day, so that the lifetime of the satellite becomes considerably shorter than it would be without this effect. Radiation pressure is also found to produce significant perturbations in the orbit of the Vanguard I satellite which remove the principal discrepancy between theory and observation in the analysis of the Vanguard I orbit [*Musen, Bailie, and Bryant*, 1960].

The terms of long period make the principal contribution to the orbit perturbations. In the present investigation we have therefore neglected the effect of the earth's shadow which has the period of the satellite's mean motion. The shadow effect does not change the nature of the perturbations, although it produces a change in their amplitudes. We have also neglected all other periodic terms having the mean anomaly of the satellite in the argument.

The vectorial method is used in the development to obtain several kinematical relations. The equations for the scalar elements are deduced from the equations for the vectorial elements. All elements except the semimajor axis contain long-period terms, but the semimajor axis is affected by short-period variations only and is therefore not subject to substantial perturbations.

Perturbations in the orbit plane. Let us designate the gravitational constant by G, the mass of the earth by M and its equatorial radius by ρ. Let $\mathbf{R}$ be the unit vector directed along the normal to the orbit plane, $\mathbf{P}$ be the unit vector directed from the center of the earth to perigee, and let $\mathbf{Q} = \mathbf{R} \times \mathbf{P}$. The position and the velocity vectors of the satellite are represented by $\mathbf{r}$ and $\mathbf{v}$, respectively; the radius vector of the satellite will be designated by r, the true anomaly by f, the mean anomaly by l; and the other elliptic elements will be designated, using the standard notations, by ω, Ω, i, e, a, n. The mean longitude of the sun on the ecliptic will be designated by λ', and the mean motion of the sun will be designated by n'. Let ϵ be the inclination of the equator to the ecliptic, $\mathbf{i}$ the unit vector directed from the center of the earth toward the vernal equinox, $\mathbf{k}$ the unit vector normal to the earth's equator, and let $\mathbf{j} = \mathbf{k} \times \mathbf{i}$.

66C

Finally, let $\mathbf{u}^0$ be the unit vector directed from the center of the earth toward the sun. Neglecting the eccentricity of the earth's orbit, we can put

$$\mathbf{u}^0 = \mathbf{i} \cos \lambda' + \mathbf{j} \cos \epsilon \sin \lambda' + \mathbf{k} \sin \epsilon \sin \lambda' \tag{1}$$

The solar radiation pressure will be

$$\mathbf{F} = F\mathbf{u}^0 \quad F < 0 \tag{2}$$

F = constant for a spherical satellite; for the satellite of nonspherical form we must make a certain assumption about the average value of F and use the average value in the development. We shall deal with the vectorial element

$$\mathbf{g} = e\mathbf{P} \tag{3}$$

and with the time variation of this vector with respect to a system of coordinates rigidly connected to the osculating orbit plane. In other words, we shall deal with the motion of $\mathbf{P}$ in this plane. For that purpose it is convenient to use Herrick's equation [*Herrick*, 1948]

$$GM\, d\mathbf{g}/dt = \Gamma \cdot \mathbf{F} \tag{4}$$

where

$$\Gamma = 2\mathbf{r}\mathbf{v} - \mathbf{v}\mathbf{r} - \mathbf{r}\cdot\mathbf{v}I \tag{5}$$

$\mathbf{r}$ and $\mathbf{v}$ in this case are given, not with respect to our inertial system, but with respect to a system rigidly connected to the osculating orbit plane. The notations $\mathbf{rv}$ and $\mathbf{vr}$ represent the dyadic products; in other words, they are the products

column vector · row vector

and I is the planar idemfactor (the planar unit matrix). Substituting

$$\mathbf{r} = \mathbf{P}r \cos f + \mathbf{Q}r \sin f \tag{6}$$

$$\mathbf{v} = -\mathbf{P}\frac{\sqrt{GM}\sin f}{\sqrt{a(1-e^2)}} + \mathbf{Q}\frac{\sqrt{GM}\,(\cos f + e)}{\sqrt{a(1-e^2)}} \tag{7}$$

$$I = \mathbf{PP} + \mathbf{QQ} \tag{8}$$

into (5), and, taking

$$GM = n^2a^3 \tag{9}$$

into consideration, we deduce

$$\begin{aligned}\Gamma = \frac{na^2}{\sqrt{1-e^2}}\Big\{&-\frac{r}{a}(e \sin f + \tfrac{1}{2}\sin 2f)\mathbf{PP} \\ &+ \frac{1}{2}\cdot\frac{r}{a}\sin 2f\mathbf{QQ} \\ &+ \left[\frac{r}{a}(\tfrac{1}{2} + e\cos f + \tfrac{1}{2}\cos 2f) + 1 - e^2\right]\mathbf{PQ} \\ &- \left[\frac{1}{2}\cdot\frac{r}{a}(1 - \cos 2f) + 1 - e^2\right]\mathbf{QP}\Big\}\end{aligned} \tag{10}$$

In order to separate the long periodic part in Γ from the short periodic one we must develop the coefficients of the dyadic products into Fourier series with respect to the mean anomaly and retain only the constant terms in this development. Using Cayley's tables, we find that the constant parts of the coefficients of $\mathbf{PQ}$ and of $\mathbf{QP}$ are equal to $\pm\frac{3}{2}(1 - e^2)$. The constant parts of the other coefficients are equal to zero. Thus, we have for the long periodic part of Γ

$$[\Gamma] = \tfrac{3}{2}na^2\sqrt{1-e^2}\,(\mathbf{PQ} - \mathbf{QP})$$

but

$$\mathbf{R} \times I = \mathbf{R} \times (\mathbf{PP} + \mathbf{QQ}) = \mathbf{QP} - \mathbf{PQ}$$

and we have a simple relation

$$[\Gamma] = -\tfrac{3}{2}na^2\sqrt{1-e^2}\,\mathbf{R} \times I \tag{11}$$

Substituting (3) and (11) into (4) we deduce for the long periodic part in $e\mathbf{P}$

$$\frac{d(e\mathbf{P})}{dt} = -\frac{3}{2}\cdot\frac{na^2\sqrt{1-e^2}}{GM}\mathbf{R} \times \mathbf{F} \tag{12}$$

We see that the time variation of $e\mathbf{P}$ is normal to the component of $\mathbf{F}$ lying in the orbit plane. The vector $\mathbf{P}$ in the moving system of coordinates is affected only by the rotation about $\mathbf{R}$. The angular velocity of rotation

$$\mathbf{R}\, d\pi/dt$$

Thus

$$\frac{d\mathbf{P}}{dt} = \frac{d\pi}{dt}\mathbf{R} \times \mathbf{P} = \frac{d\pi}{dt}\mathbf{Q} \tag{13}$$

It follows from (12) and (13)

$$\frac{de}{dt}\mathbf{P} + e\frac{d\pi}{dt}\mathbf{Q} = -\frac{3}{2}\cdot\frac{na^2\sqrt{1-e^2}}{GM}\mathbf{R} \times \mathbf{F}$$

or

$$\frac{de}{dt} = +\frac{3}{2}\cdot\frac{na^2\sqrt{1-e^2}}{GM}\mathbf{Q}\cdot\mathbf{F} \tag{14}$$

$$e\frac{d\pi}{dt} = -\frac{3}{2}\cdot\frac{na^2\sqrt{1-e^2}}{GM}\mathbf{P}\cdot\mathbf{F} \tag{15}$$

Substituting

$$\begin{aligned}\mathbf{P} = \mathbf{i}[&+\cos^2(i/2)\cos(\omega+\Omega) \\ &+\sin^2(i/2)\cos(\omega-\Omega)] \\ &+\mathbf{j}[+\cos^2(i/2)\sin(\omega+\Omega) \\ &-\sin^2(i/2)\sin(\omega-\Omega)] \\ &+\mathbf{k}\sin i\sin\omega\end{aligned} \tag{16}$$

into (15) and

$$\begin{aligned}\mathbf{Q} = \mathbf{i}[&-\cos^2(i/2)\sin(\omega+\Omega) \\ &-\sin^2(i/2)\sin(\omega-\Omega)] \\ &+\mathbf{j}[+\cos^2(i/2)\cos(\omega+\Omega) \\ &-\sin^2(i/2)\cos(\omega-\Omega)] \\ &+\mathbf{k}\sin i\cos\omega\end{aligned} \tag{17}$$

into (14), and taking (1)–(2) into account, we obtain

$$\begin{aligned}\frac{de}{dt} = -\frac{3F}{2GM}\cdot na^2\sqrt{1-e^2} \cdot\{&+\cos^2(i/2)\sin^2(\epsilon/2)\sin(\omega+\Omega+\lambda') \\ &+\cos^2(i/2)\cos^2(\epsilon/2)\sin(\omega+\Omega-\lambda') \\ &+\sin^2(i/2)\cos^2(\epsilon/2)\sin(\omega-\Omega+\lambda') \\ &+\sin^2(i/2)\sin^2(\epsilon/2)\sin(\omega-\Omega-\lambda') \\ &-\tfrac{1}{2}\sin i\sin\epsilon\sin(\omega+\lambda') \\ &+\tfrac{1}{2}\sin i\sin\epsilon\sin(\omega-\lambda')\}\end{aligned} \tag{18}$$

and

$$\begin{aligned}e\frac{d\pi}{dt} = -\frac{3F}{2GM}\cdot na^2\sqrt{1-e^2} \cdot\{&+\cos^2(i/2)\sin^2(\epsilon/2)\cos(\omega+\Omega+\lambda') \\ &+\cos^2(i/2)\cos^2(\epsilon/2)\cos(\omega+\Omega-\lambda') \\ &+\sin^2(i/2)\cos^2(\epsilon/2)\cos(\omega-\Omega+\lambda') \\ &+\sin^2(i/2)\sin^2(\epsilon/2)\cos(\omega-\Omega-\lambda') \\ &-\tfrac{1}{2}\sin i\sin\epsilon\cos(\omega+\lambda') \\ &+\tfrac{1}{2}\sin i\sin\epsilon\cos(\omega-\lambda')\}\end{aligned} \tag{19}$$

Perturbations in the position of the orbit plane. The angular velocity of rotation of the osculating orbit plane, considered as a rigid body,

$$\psi = \frac{na^2}{\sqrt{1-e^2}}\cdot\frac{\mathbf{r}}{a}\cdot\frac{\mathbf{R}\cdot\mathbf{F}}{GM} \tag{20}$$

The part of $\mathbf{r}$ independent of l is

$$-\tfrac{3}{2}\mathbf{P}ae$$

and, consequently, the long periodic part of ψ is

$$[\psi] = -\tfrac{3}{2}\cdot\frac{na^2e}{\sqrt{1-e^2}}\cdot\frac{\mathbf{PR}\cdot\mathbf{F}}{GM} \tag{21}$$

The variation of $\mathbf{R}$ is caused only by the rotation of the osculating orbit plane, and we have then for the long periodic part in $\mathbf{R}$

$$d\mathbf{R}/dt = -\mathbf{R} \times [\psi]$$

or

$$\frac{d\mathbf{R}}{dt} = +\frac{3}{2}\cdot\frac{na^2e\mathbf{QR}\cdot\mathbf{F}}{GM\sqrt{1-e^2}} \tag{22}$$

and we conclude that the long-period time variation of $\mathbf{R}$, caused by the radiation pressure, consists of the rotation of $\mathbf{R}$ about $\mathbf{P}$ with the angular velocity proportional to the cosine of the angle between the direction to the sun and $\mathbf{R}$.

This time variation is zero at the moment when $\mathbf{R}$ is normal to $\mathbf{u}^0$, in particular when $\mathbf{R}$ is normal to the ecliptic. From equation (22) we can also deduce the equations for i and Ω. The angular velocity $[\psi]$ can be decomposed into the geometrical sum of (1) the angular velocity di/dt of the rotation of the orbit plane around the

line of nodes and (2) the angular velocity $\sin i\omega\, d\Omega/dt$ of the rotation around the vector $\mathbf{P}\sin\omega + Q\cos\omega$. We have for the long-period variation in i and Ω, taking (21) into account,

$$(\mathbf{P}\cos\omega - \mathbf{Q}\sin\omega)\frac{di}{dt} + (\mathbf{P}\sin\omega + \mathbf{Q}\cos\omega)\cdot\sin i\,\frac{d\Omega}{dt} = -\frac{3}{2}\cdot\frac{na^2e}{\sqrt{1-e^2}}\cdot\frac{\mathbf{PR}\cdot\mathbf{F}}{GM}$$

We deduce from this last equation, after the scalar multiplication by $\mathbf{P}\cos\omega - \mathbf{Q}\sin\omega$ and by $\mathbf{P}\sin\omega + \mathbf{Q}\cos\omega$ and taking $(\mathbf{P}\cos\omega - \mathbf{Q}\sin\omega)\cdot\mathbf{k} = 0$ and $\mathbf{P}\cdot\mathbf{k} = \sin i\sin\omega$ into consideration,

$$\frac{di}{dt} = -\frac{3}{2}\cdot\frac{na^2e\mathbf{R}\cdot\mathbf{F}\cos\omega}{GM\sqrt{1-e^2}} \quad (23)$$

$$\sin i\,\frac{d\Omega}{dt} = -\frac{3}{2}\cdot\frac{na^2e\mathbf{R}\cdot\mathbf{F}\sin\omega}{GM\sqrt{1-e^2}} \quad (24)$$

Taking

$$\mathbf{R} = \mathbf{i}\sin i\sin\Omega - \mathbf{j}\sin i\cos\Omega + \mathbf{k}\cos i \quad (25)$$

and (1)–(2) into consideration, we have after some easy trigonometrical transformations

$$\begin{aligned}\frac{di}{dt} = -\frac{3}{4}\cdot\frac{Fna^2e}{GM\sqrt{1-e^2}} &\cdot\{+\sin i\sin^2(\epsilon/2)\sin(\omega+\Omega+\lambda')\\ &-\sin i\sin^2(\epsilon/2)\sin(\omega-\Omega-\lambda')\\ &+\sin i\cos^2(\epsilon/2)\sin(\omega+\Omega-\lambda')\\ &-\sin i\cos^2(\epsilon/2)\sin(\omega-\Omega+\lambda')\\ &+\cos i\sin\epsilon\sin(\omega+\lambda')\\ &-\cos i\sin\epsilon\sin(\omega-\lambda')\}\end{aligned} \quad (26)$$

$$\begin{aligned}\sin i\,\frac{d\Omega}{dt} = +\frac{3}{4}\cdot\frac{Fna^2e}{GM\sqrt{1-e^2}} &\cdot\{+\sin i\sin^2(\epsilon/2)\cos(\omega+\Omega+\lambda')\\ &-\sin i\sin^2(\epsilon/2)\cos(\omega-\Omega-\lambda')\\ &+\sin i\cos^2(\epsilon/2)\cos(\omega+\Omega-\lambda')\\ &-\sin i\cos^2(\epsilon/2)\cos(\omega-\Omega+\lambda')\\ &+\cos i\sin\epsilon\cos(\omega+\lambda')\\ &-\cos i\sin\epsilon\cos(\omega-\lambda')\}\end{aligned} \quad (27)$$

These equations could also be obtained in the standard way.

The integration problem. If we assume that we do not have sharp resonance conditions, the first-order perturbations can easily be obtained by the integration of equations (18), (19), (26), and (27), assuming that the elements in the right-hand sides are replaced by their mean values. The oblateness of the earth produces the secular motions of the node and of the argument of the perigee, for which we have

$$\omega = \omega_0 + \frac{6k_2\rho^2n\left(1 - \frac{5}{4}\sin^2 i\right)}{a^2(1-e^2)^2}(t - t_0)$$

$$\Omega = \Omega_0 - \frac{3k_2\rho^2n\cos i}{a^2(1-e^2)^2}(t - t_0)$$

The resonance case. The resonance case deserves special attention and a special treatment. The most interesting resonance occurs when the perigee of the satellite closely follows the sun, i.e., when the critical argument, in the terminology of celestial mechanics, is

$$\omega + \Omega - \lambda'$$

The term with this argument is the most influential one in the development of the radiation perturbations in the case of Vanguard I.

In order to simplify the exposition we shall adopt the system of units in use at the Vanguard Computer Center in the computation of the general oblateness perturbations. We put

$$G = 1 \qquad M = 1 \qquad \rho = 1$$

and we use the system of coordinates rotating uniformly in the equatorial plane with angular velocity equal to the mean motion of the sun in the ecliptic. In our exposition we follow the line of thought laid down by Brown in his planetary theory [*Brown and Shook*, 1933], modifying the form of the disturbing function to serve our purpose, and, as Brown did in the

planetary case, we will reduce our problem to an equation similar in form to the equation of the motion of a mathematical pendulum. The modified canonical set of Poincaré

$$x_1 = \sqrt{a}\,(1 - \sqrt{1 - e^2}) \qquad y_1 = -\omega - \Omega + \lambda'$$
$$x_2 = \sqrt{a} \qquad y_2 = l + \omega + \Omega - \lambda' \tag{28}$$
$$x_3 = \sqrt{a(1 - e^2)}\,(1 - \cos i) \qquad y_3 = -\Omega + \lambda'$$

is especially convenient here. The corresponding Hamiltonian function consists of several parts:

1. The part

$$\frac{1}{2a}$$

is contributed by the elliptic motion of the satellite.

2. The part

$$n'\sqrt{a(1 - e^2)}\cos i$$

is generated by the motion of the sun.

3. The secular part

$$\frac{k_2(1 - \frac{3}{2}\sin^2 i)}{a^3(1 - e^2)^{3/2}}$$

is produced by the oblateness of the earth.

4. The periodic part

$$-\tfrac{3}{2} Fae \cos^2(i/2)\cos^2(\epsilon/2) \cdot \cos(\omega + \Omega - \lambda')$$

comes from the development of the disturbing potential $-\mathbf{F}\cdot\mathbf{r}$ into a trigonometric series.

Thus, we have

$$H = \frac{1}{2a} + n'\sqrt{a(1 - e^2)}\cos i + \frac{k_2(1 - \frac{3}{2}\sin^2 i)}{a^3(1 - e^2)^{3/2}} - \tfrac{3}{2}Fae\cos^2(i/2) \cdot \cos^2(\epsilon/2)\cos(\omega + \Omega - \lambda') \tag{29}$$

or, taking (28) into consideration,

$$H = \frac{1}{2x_2^2} + n'(x_2 - x_1 - x_3) + \frac{k_2}{x_2^3}\left[\frac{1}{(x_2 - x_1)^3} - \frac{3x_3}{(x_2 - x_1)^4} + \frac{3x_3^2}{2(x_2 - x_1)^5}\right] + k_2K\cos y_1 \tag{30}$$

where we put

$$k_2K = -\tfrac{3}{2}Fae\cos^2(i/2)\cos^2(\epsilon/2) > 0$$

We have

$$\frac{dx_1}{dt} = +\frac{\partial H}{\partial y_1} \qquad \frac{dy_1}{dt} = -\frac{\partial H}{\partial x_1} \tag{31}$$

The arguments y_2 and y_3 are not present in (30), and from

$$\frac{dx_2}{dt} = +\frac{\partial H}{\partial y_2} = 0 \qquad \frac{dx_3}{dt} = +\frac{\partial H}{\partial y_3} = 0$$

it follows that

$$x_2 = \alpha_2 = \text{constant}$$
$$x_3 = \alpha_3 = \text{constant}$$

and we also have the integral of Jacobi

$$H = h$$

If the satellite is approximately of the size and the form of Vanguard I, the perturbations in the eccentricity are small, even if the resonance is a sharp one. The ratio F/k_2 in this case is also small, and the coefficient K might be considered to be invariable. Let us introduce a new variable ξ instead of x_1, and a new independent variable T instead of t, by putting

$$x_1 = \alpha_1 + \sqrt{k_2}\,\xi \tag{32}$$
$$\sqrt{k_2}\,dt = dT \tag{33}$$

where α_1 is a constant to be determined.

Equations 31, if transformed to the new variables, becomes

$$\frac{d\xi}{dT} = +\frac{\partial W}{\partial y_1} \qquad \frac{dy_1}{dT} = -\frac{\partial W}{\partial \xi} \tag{34}$$

where

$$W = \frac{1}{\sqrt{k_2}}\left(k_2\frac{\partial \phi}{\partial \alpha_1} - n'\right)\xi + k_2\frac{\partial^2\phi}{\partial \alpha_1^2}\xi^2 + K\cos y_1 \tag{35}$$

and

$$\phi = \frac{1}{\alpha_2^3}\left[\frac{1}{(\alpha_2-\alpha_1)^3} - \frac{3\alpha_3}{(\alpha_2-\alpha_1)^4} + \frac{3\cdot\alpha_3^2}{2(\alpha_2-\alpha_1)^5}\right] \qquad (36)$$

Let us determine the constant α_1 in such a way that the condition

$$n' - k_2\frac{\partial\phi}{\partial\alpha_1} = 0 \qquad (37)$$

is satisfied. This last equation says that in the first approximation the argument y_1 does not have a secular motion. In other words, to assert (27) is the same as to assert the existence of the resonant conditions. Equation (35) becomes

$$W = \tfrac{1}{2}k_2\frac{\partial^2\phi}{\partial\alpha_1^2}\xi^2 + K\cos y_1 \qquad (35')$$

and

$$\phi''_{\alpha_1^2} = \frac{\partial^2\phi}{\partial\alpha_1^2} > 0$$

for moderate inclinations. We deduce from (34) and (35′)

$$\frac{d\xi}{dT} = -K\sin y_1 \qquad (38)$$

$$\frac{dy_1}{dT} = -k_2\phi''_{\alpha_1^2}\cdot\xi \qquad (39)$$

From these two last equations we obtain

$$\frac{d^2y_1}{dT^2} = k_2\phi''_{\alpha_1^2}\cdot K\sin y_1 \qquad (40)$$

The integration of this equation is given in treatises on analytical dynamics, and it is not necessary to reproduce it here. The following three cases exist

1. The critical argument y_1 oscillates around 180°; we have the case of libration.
2. The critical argument has a secular motion.
3. The critical argument approaches zero asymptotically.

Despite the fact that the libration of the critical argument may be large, the perturbations in x_1 are always small.

In the case of libration we have

$$\cos\tfrac{1}{2}y_1 = k\,sn\,(\nu T, k) = k\,sn\,(k_2\sqrt{K\phi''_{\alpha_1^2}}\,t, k) \qquad (41)$$

where

$$\nu^2 = k_2K\phi''_{\alpha_1^2}, \qquad (42)$$

and $0 < k < 1$ is the constant of integration. We deduce

$$\frac{dy_1}{dT} = -2k\nu\,cn(\nu T, k) \qquad (43)$$

and, taking (32), (33), (42), and (43) into consideration, we obtain

$$x_1 = \alpha_1 + 2k\sqrt{\frac{K}{\phi''_{\alpha_1^2}}}\,cn(\nu T, k)$$

If the motion of y_1 is progressive, i.e., if y_1 has secular motion, then we deduce

$$y_1 = \pi - 2am(\nu T, k) = \pi - 2am\left(\frac{k_2}{k}\sqrt{K\phi''_{\alpha_1^2}}\,t, k\right)$$

and

$$\nu^2k^2 = k_2K\phi_{\alpha_1^2}''$$

$$x_1 = \alpha_1 + \frac{2}{k}\sqrt{\frac{K}{\phi''_{\alpha_1^2}}}\,dn(\nu T, k)$$

And, finally, in the case of the asymptotic approach,

$$tg\,\frac{y_1}{4} = Ce^{-\nu T}, \quad \nu^2 = k_2K\phi''_{\alpha_1^2}$$

$$y_1 \to 0, \qquad \xi \to 0, \qquad x_1 \to \alpha_1$$

if $T \to +\infty$.

Acknowledgment. The author wishes to express his gratitude to Drs. Robert Jastrow and John A. O'Keefe of the Theoretical Division of Goddard Space Flight Center, who suggested the investigation of this effect and with whom the author had several valuable discussions.

References

Brown, E. W., and C. A. Shook, *Planetary Theory*, pp. 216–249, Cambridge, 1933.

Herrick, S., *Publs. Astron. Soc. Pacific, 60*, 321, 1948.

Musen, P., A. Bailie, and R. Bryant, Perturbations in perigee height of Vanguard I, *Science, 131*, 935–936, 1960.

(Manuscript received March 4, 1960.)